CONSTRAINTS ON THREE-NEUTRINO MIXING
FROM ATMOSPHERIC AND REACTOR DATA

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ABSTRACT

Observations of atmospheric neutrinos are usually analyzed using the simplifying approximation that either $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_e \leftrightarrow \nu_\mu$ two-flavor mixing is relevant. Here we instead consider the data using the simplifying approximation that only one neutrino mass scale is relevant. This approximation is the minimal three-flavor notation that includes the two relevant two-flavor approximations. The constraints in the parameter space orthogonal to the usual, two-flavor analyses are studied.

In recent years, large water Cherenkov detectors located deep underground have been able to provide statistically significant measurements of the flux of atmospheric neutrinos [1, 2]. They have found that the flavor content of the flux differs from expectations [3]. In particular, Kamiokande and IMB have found the ratio of ν_μ/ν_e in their fully contained events to be 0.60 $\pm$ 0.09 and 0.55 $\pm$ 0.09 , respectively, of what they expected. Smaller, but differently constituted, detectors have provided less statistically significant results that are consistent with these observations [4, 5]. This discrepancy could be caused by neutrino mixing (see e.g. [6, 7]).

Atmospheric neutrinos involve measurably distinct fluxes of more than one flavor of neutrino. In principle, they depend on the full possible range of neutrino mixing parameter space: two mass-squared difference scales, three mixing angles and a CP violating phase. In practice, the present data are rather crude so that the presence of any nonzero neutrino oscillations parameters is not yet certain. Thus the observations have (generally) only been analyzed in the two-flavor approximation where the number of parameters is minimal: one mass scale and one mixing angle. However there are good reasons for going beyond the two-flavor approximation.

One reason for going beyond the two-flavor approximation is that the data presently indicate that at least one of the mixing angles is rather large [8]. Also, the effective ν_e mixing is sometimes enhanced for atmospheric neutrinos by matter effects. As is well known, the two-flavor approximation can be quite poor when a mixing angle, either a vacuum angle or a matter enhanced angle [9, 10, 11], is large.

Neutrino masses are small because of the structure of the standard model

[12]. Neutrino mixings can be predicted in some extensions of the standard model. At present, the most attractive extension of the standard model is the SO(10) Grand Unified Theory. In this model, with "minimal" particle content, the neutrino mixing is calculable in terms of the quark mixing—however both small and large mixing solutions exist [13]. Additional, unpredictable contributions are generally also present. Thus theoretical arguments can not exclude any values of the vacuum mixing angles.

A second reason for going beyond the two-flavor approximation is to understand how to test the neutrino oscillation explanation of the measurements. The continuing experiments and many new experiments presently under construction [14] will improve our knowledge of the atmospheric neutrino flux. However uncertainties in the calculation of the atmospheric neutrinos production limit the ability of these experiments to constrain neutrino mixing parameters. Consequently many new experiments using neutrinos produced at reactors [15] and at accelerators [16] are being planned to study the specific neutrino parameters believed to be relevant for the atmospheric neutrino discrepancy. Hence it is important to know exactly what those parameters are.

A third reason for going beyond the two-flavor approximation is because the contained atmospheric events involve more than one flavor of neutrino. Hence they can be explained by two different types of two-flavor mixing, either $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_e \leftrightarrow \nu_\mu$. Analyses of the data are routinely done in both of these approximations. However, as is generally realized but seldom discussed, there is a continuous parameter space between these two limits. The purpose of this article is to clarify and explore this intermediate region

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between the two relevant two-flavor approximations.

Here we compromise and examine the data in a simplified three-flavor formalism. We *assume* that one of the mass-squared scales is less than 3×10^{-5} eV^2 and hence irrelevant since the associated oscillation wavelength is longer than the longest propagations lengths of the current experiments (the diameter of the Earth) [17]. Then there is only one mass-squared scale and two mixing angles which are relevant. A heuristic way to express this is to just set

$$m_1 = m_2 = 0, \quad m_3 > 0. \tag{1}$$

Then the remaining two mixing angles can be thought of as defining the amount of ν_e and ν_μ in the one massive state (the amount of ν_τ in this state is fixed by unitarity).

The parameterization of the mixing

$$|\nu_\alpha> = U_{\alpha i}|\nu_i> \tag{2}$$

between the flavor eigenstates, $\alpha = e, \mu, \tau$, and the mass eigenstates, $i = 1, 2, 3$, is here chosen to be

$$U = \begin{bmatrix} 0 & \cos\phi & \sin\phi \\ -\cos\psi & -\sin\psi\sin\phi & \sin\psi\cos\phi \\ \sin\psi & -\cos\psi\sin\phi & \cos\psi\cos\phi \end{bmatrix} \tag{3}$$

where ϕ and ψ are the mixing angles. This parameterization is chosen such that matter effects [18] are straightforward (for a general review of matter effects see [10]). In a matter background, ψ is unchanged but the effective ϕ

is given by

$$\sin^2 2\phi_m = \frac{(m_3^2 \sin 2\phi)^2}{(A - m_3^2 \cos 2\phi)^2 + (m_3^2 \sin 2\phi)^2} \quad . \tag{4}$$

and the effective mass eigenstates are

$$M_1^2 = 0 \tag{5}$$

$$M_{3,2}^2 = \frac{1}{2}[(m_3^2 + A) \pm \sqrt{(A - m_3^2 \cos 2\phi)^2 + (m_3^2 \sin 2\phi)^2}]$$

where the i=2 state is associated with the minus sign [19]. Here A is the induced mass-squared from the electron background,

$$A = 2\sqrt{2}G_F(Y_e\rho/m_u)E \tag{6}$$

$$= 3.8 \times 10^{-4} eV^2 \left(\frac{Y_e\rho}{2.5g/cm^3}\right)\left(\frac{E}{1GeV}\right)$$

with G_F as Fermi's constant, Y_e is the number of electrons per nucleon, ρ is the density, m_u is the nucleon mass, and E the neutrino energy. For antineutrinos, $A \to -A$.

To illustrate the physical implications of this parametrization, we give the relevant oscillations probabilities for a constant density medium.

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\phi_m[1 - \cos(\beta_3 - \beta_2)]$$

$$P(\nu_\mu \to \nu_e) = \frac{1}{2}\sin^2\psi \sin^2 2\phi_m[1 - \cos(\beta_3 - \beta_2)] \tag{7}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{2}\{\sin^2 2\psi[1 - \sin^2\phi_m \cos(\beta_2 - \beta_1) - \cos^2\phi_m \cos(\beta_3 - \beta_1)]$$

$$+ \sin^4\psi \sin^2 2\phi_m[1 - \cos(\beta_3 - \beta_2)]\}$$

Here the dynamical phase acquired by a neutrino mass eigenstate which propagates for a time t is

$$\beta_i \equiv \frac{M_i^2 t}{2E} \tag{8}$$

Unitarity and time reversal symmetry [20] can be used to obtain the other oscillation probabilities from those above. For more complicated density distributions, the procedure for calculating the probabilities is straightforward (see e.g. [10]).

In the two-flavor *vacuum* approximation, neutrino oscillation effects are symmetric for mixing angles in the ranges 0 to $\pi/4$ and $\pi/4$ to $\pi/2$. However when there are three flavors (and also when matter effects are relevant) there is no symmetry between these two ranges. Thus we use limits where ϕ and ψ explicitly range between 0 and $\pi/2$, or, equivalently, the sines of these angles range between 0 and 1. This covers the full allowed range for these parameters, without any redundancy.

When one of the mixing angles is at the limit of its range, then one of the neutrino flavors decouples and the approximate three-flavor notation (Eq. (3)) reduces to a two-flavor description in terms of the remaining mixing angle (see Table). All possible two-flavor approximations are included. However since there are only two mixing angles, the three possible types of two-flavor approximation are not fully independent. This just follows from the assumption of only one relevant mass scale.

In this notation the neutrino parameter space has three dimensions: two mixing angles and the mass-squared. The conventional two-flavor plots of mass-squared versus a mixing angle correspond to one of the two dimensional surfaces of this three dimensional parameter space. To complement this usual approach, we here show plots (Figs. (1) and (2)) at fixed mass-squared. These plots of one mixing angle versus the other mixing angle show cross sections of the parameter space which are orthogonal to the usual two-flavor

plots.

The region between the dotted contours in Fig. (2) is excluded by reactor measurements [21]. Reactor experiments are purely ν_e disappearance experiments. As can be seen from Eqs. (7), the expression for $P(\nu_e \rightarrow \nu_e)$ is always equivalent to the two-flavor approximation. Thus no additional calculations are necessary. The mass-squared value in Fig. (1) is below the present two-flavor limits from reactor experiments, so there are no constraints. For the fixed mass-squared of Fig. (2), the mixing angle is a fixed value and the dotted contour is a vertical line.

Atmospheric neutrinos involve more than one flavor, so the three-flavor effects are significant and the constraint contours must be calculated accordingly. Following the accepted two-flavor analyses, we calculate oscillation constraints using ratios of atmospheric neutrino flux measurements. This is to cancel out the large errors inherent in modeling the production of atmospheric neutrinos. Two different ratios are used: R_ν, the ratio of ν_μ and ν_e fluxes in fully contained events ($< E > \sim 0.8$ GeV); and R_μ, the ratio of the fluxes of upward going muons that stop in the detector ($< E > \sim 10$ GeV) to those that go completely through the detector ($< E > \sim 100$ GeV). For both of these ratios, we do not perform a chi-squared fit to the energy and angular distributions of the data (which would require detailed knowledge of the detector resolutions, efficiencies, thresholds, scattering cross sections, etc), but instead just fit to the total flux ratios. For R_ν we use Kamiokande's value, for R_μ we use IMB's value with an extra 7% systematic uncertainty [8]. The ratio of ν_μ/ν_e at production is taken to be 2.0, the ratio of $\bar{\nu}/\nu$ in the detected events is taken to be 0.4, the oscillation probabilities are aver-

aged over the approximate energy distributions given in ref. [8] and a 1/L neutrino path length distribution. The contours shown in the Figures are all 90% confidence levels.

In the Figures, the parameter regions allowed by all the constraints are shaded. The parameter region preferred by R_ν ranges continuously from the left boundary ($\nu_\mu \leftrightarrow \nu_\tau$ mixing) to the upper boundary ($\nu_e \leftrightarrow \nu_\mu$ mixing). The parameter region excluded by R_μ connects only to the left boundary ($\nu_\mu \leftrightarrow \nu_\tau$ mixing). The values on the boundaries are in rough agreement with previous two-flavor analyses. The constraint by R_μ in the $\nu_e \leftrightarrow \nu_\mu$ two-flavor approximation has not been discussed previously by the experimental groups. That limiting constraint is sensitive to the ratio of ν_μ/ν_e at high energies, and it vanishes for the "large" value of this ratio used here.

Somewhat below the mass-squared value of Fig. (1), the average oscillation wavelength becomes larger than the radius of the Earth and all of the constraints vanish. The constraints from R_μ are smaller at the larger mass-squared of Fig. (2), and quickly vanish with increasing mass-squared, because then both the stopping and through-going muon fluxes are equally reduced by mixing. The constraints from R_ν are constant with increasing mass-squared.

The constraints in the Figures were also computed without the matter background, to study the importance of this effect. For the R_ν contours, the value of $\sin^2 \phi$ near the center of the Figures would be reduced by 50% if matter effects were neglected, while the values on the boundaries would essentially remain unchanged. For the R_μ contours, the excluded area would be reduced by a third to a half if matter effects were neglected. Thus the

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matter background is quantitatively quite important for the atmospheric neutrino contours.

The Figures illustrate some general features of three-flavor effects. For example, note that the contours from the reactor experiments and R_μ are mostly vertical and horizontal (on the left) lines, respectively, while the R_ν contours are sloping. Thus the overlap between the two excluded regions and the preferred region is rather imperfect. An enlargement of these exclusion regions could result in a situation where, at a given mass-squared, the two two-flavor approximations showed no allowed region while there still was some allowed region in a three-flavor analyses (toward the upper left corner of the Figures). Thus we can conclude that to definitively exclude the possibility of an allowed region, the constraint contours may have to extend well beyond what was indicated by the two-flavor analyses.

Appearance experiments are typically far more sensitive to neutrino oscillations than disappearance experiments. A long-baseline ν_μ to ν_e appearance experiment is experimentally feasible, and would have a large overlap with the three-flavor allowed region from contained atmospheric events. In addition, matter effects enhance the sensitivity of such experiments, as noted previously [11].

In summary, the one mass scale approximation gives a simple, reasonable, three-flavor notation for analyzing atmospheric and reactor neutrino observations. There are then only three neutrino parameters: one mass scale and two mixing angles. The parametrization here smoothly describes the transition between the the $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_e \leftrightarrow \nu_\mu$ two-flavor approximations usually used to analyze the data. To complement the usual calculations,

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the constraints at fixed mass squared have been computed from atmospheric neutrino contained events, atmospheric neutrino induced muons, and reactor neutrinos. The matter background of the Earth has been included, and has a quantitatively important effect on the contours. A three-flavor notation may be crucial for determining atmospheric neutrino oscillations.

Acknowledgements

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Table. The two mixing angles, ψ and ϕ, range between 0 and $\pi/2$. When one of these mixing angle is at the limit of its range, this three-flavor notation (Eq. (3)) reduces to a two-flavor approximation. The parameter limits and corresponding equivalent two-flavor approximation are given below.

Angle limit	Equivalent two-flavor mixing
$\sin^2 \psi = 1.0$	$\nu_e \leftrightarrow \nu_\mu$
$\sin^2 \psi = 0.0$	$\nu_e \leftrightarrow \nu_\tau$
$\sin^2 \phi = 0.0$	$\nu_\mu \leftrightarrow \nu_\tau$
$\sin^2 \phi = 1.0$	no oscillations

Figures. Plots of $\sin^2 \psi$ versus $\sin^2 \phi$ at constant mass-squared. The dashed lines show the excluded region from the ratio of stopping/through-going atmospheric neutrino induced muons. The dotted lines show the excluded region from reactor experiments. The solid line shows the preferred region from contained atmospheric neutrino observations. The allowed region is shaded. (1) $m^2 = 3 \times 10^{-3}$ eV^2, (2) $m^2 = 3 \times 10^{-2}$ eV^2.

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